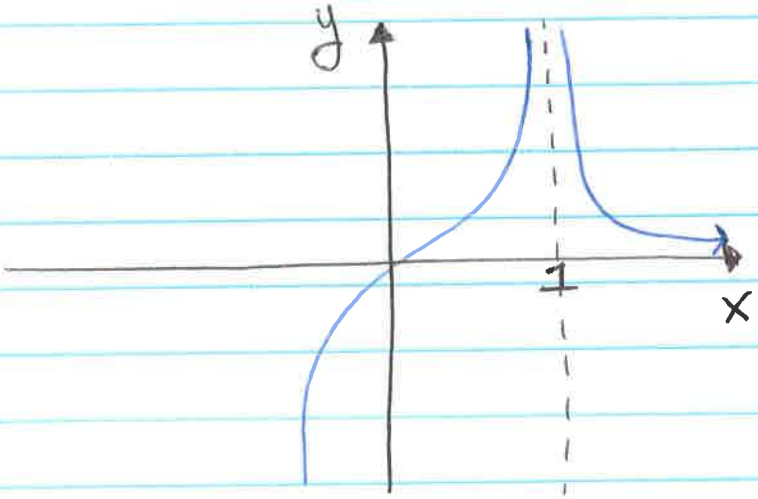
Section 1.5

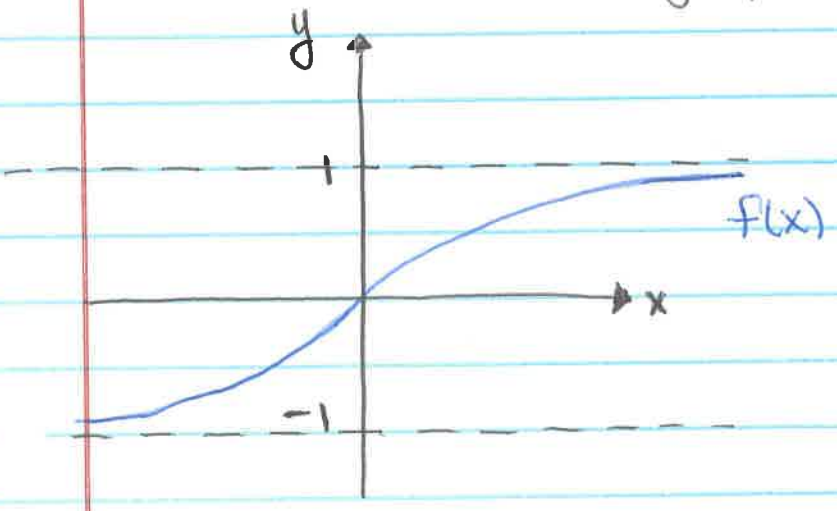
Limits Involving Infinity:

Asymptotes



$$\lim_{x \to 1} f(x) = \infty$$

"Vertical Asymptote"



$$\lim_{x \to \infty} f(x) = 1$$

$$\lim_{x \to -\infty} f(x) = -1$$

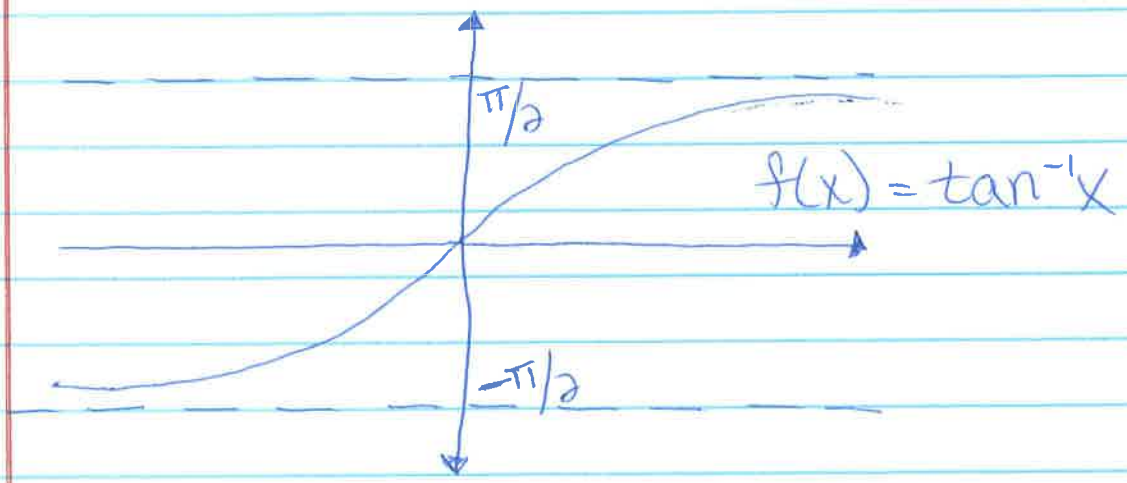
Infinite Limit Rules

① $\lim_{x \to \pm\infty} \frac{1}{x^t} = 0$

where $t > 0$.

$\lim_{x \to \infty} \tan^{-1} x = \pi/2$

$\lim_{x \to -\infty} \tan^{-1} x = -\pi/2$



How to find Horizontal Asymptotes

* If the degree of the numerator is higher, then $\lim_{x \to \infty} f(x) = -\infty$ or ∞

* If the degree of denominator is higher, then $\lim_{x \to \infty} f(x) = 0$

* If the degrees are the same, then $\lim_{x \to \infty} f(x)$ is equal to the leading Coefficient of the numerator

divided by the leading coefficient of the denominator.

Example 1:

Determine each limit with a number, ∞, $-\infty$, or DNE

① $\lim_{x \to 3^+} \frac{x+2}{x-3}$ $\frac{+}{+}$

$\lim_{x \to 3^+} \frac{x+2}{x-3} = \infty$

② $\lim_{x \to \infty} \frac{x^3 + 5x}{2x^3 - x^0 + 4}$

"Degrees are equal"

$$\lim_{x \to \infty} \frac{x^3 + 5x}{2x^3 - x^2 + 4}$$

Horizontal Asymptote

Degrees are equal so Limit equals a number.

$$\text{So } \lim_{x \to \infty} \frac{1x^3 + 5x}{2x^3 - x^2 + 4} = \boxed{\frac{1}{2}}$$

③ $$\lim_{x \to \infty} \frac{3x + 5}{x^2 + 1}$$

$x \to \infty$ Tells us to look For a horizontal Asymptote.

Degree in denominator is greater, so $\lim_{x \to \infty} f(x) = 0$

Thus, $$\lim_{x \to \infty} \frac{3x + 5}{x^2 + 1} = 0$$

pg 118 # 1-4 all, 8-10 all, 25-29 odd 39-43 odd